1. Method to calculate the envelope field

The upper tropospheric quasi-stationary wave (QSW) is calculated from the anomalous meridional wind v' at 300 hPa. This quantity we seek is a stationary or slowly propagating wave signal and therefore a 15 day lowpass filter is applied to the raw meridional wind, v, to remove the faster propagating wave signals. The resulting filtered data are subsequently denoted as $\tilde{v}$. A climatological annual cycle $\overline{\tilde{v}}$ is then calculated and removed to produce the resulting anomalous meridional wind ($v' = \tilde{v} - \overline{\tilde{v}}$) which is then used to calculate the QSW.

The QSW is assumed to consist of the form

$$v'(\lambda) = A(\lambda)\, C(\lambda) \ ,$$

where λ is longitude, C is the so-called carrier wave and A is the slowly varying amplitude. The carrier wave is a simple sine function $C = \sin(s\,\lambda)$ with wavenumber s. While C oscillates between positive and negative values, A is non-negative everywhere and varies on a much larger spatial scale than C. In the following, we refer to A as the envelope of the wave. In reality, the wave consists not of a carrier wave with only one specific wavenumber s, but a range of wavenumbers ($s_1 \leqslant s \leqslant s_2$). With $v'(\lambda)$ given and assuming C consists of a specific range of relevant wavenumbers (typically $s_1 \approx 4$ to $s_2 \approx 8$ in the midlatitudes), we can calculate the envelope of the wave A.

For this envelope reconstruction we use the method of Zimin et al. (2003), but instead of using a fixed wavenumber range, we choose a latitude-dependent wavenumber range (as in Wolf and Wirth, 2017). The latitude-dependence is based on the cosine decay of the main contribution of the power spectra of v'. The power spectra of v' and the consequent choice of the latitude-dependent wavenumber range (and its relation to classic barotropic Rossby wave theory) is discussed in appendix A.

As an illustration of the method, we show the individual steps of the QSW calculation applied to 09.08.2003 (Fig. 1). From the meridional wind (shading in Fig. 1a) or geopotential (contour lines) one can by eye easily identify a clear wave pattern in the zonal direction (Fig. 1a) which becomes even more clearly visible after applying the 15 day lowpass filter (Fig. 1b). The envelope reconstruction captures the region exposed to this wave pattern (Fig. 1c). However, the envelope mainly shows high values in the center of the wave signal and strongly decreasing values towards the meridional edges of the wave. For further analysis we therefore usually applied a Hann filter to the adjacent 7.5° latitude to slightly increase the meridional dimension of the wave envelope, but this does not qualitatively change the results and is therefore not included in this data.

A. Choice of the latitude-dependent wavenumber range

In this appendix we explain the choice of the latitude-dependent wavenumber range, which is used to calculate the QSWs. The latitude dependence is based on a cosine decay of the wavenumber range with increasing latitude.

The cosine decay is defined in such a way as to capture the main contribution of the power spectra of the wind field used to calculate the envelope field (as in Wolf and Wirth (2017) for the envelope reconstruction of propagating wave packets). The underlying power spectra with the chosen wavenumber range can be seen in Fig. 2. The black lines show the latitude dependent wavenumber range used to reconstruct the envelope of the QSW. This wavenumber range includes the wavenumbers derived from the barotropic Rossby wave dispersion relation

$$\omega = u\,k - \frac{(\beta - u_{\mathrm{yy}})\ k}{k^2 + l^2} \ , \tag{1}$$

where u is the zonal background wind, k and l the zonal and meridional wavenumber, the index y the meridional derivative and $\beta = f_{\mathrm{y}}$ the meridional gradient of the Coriolis parameter. We can solve

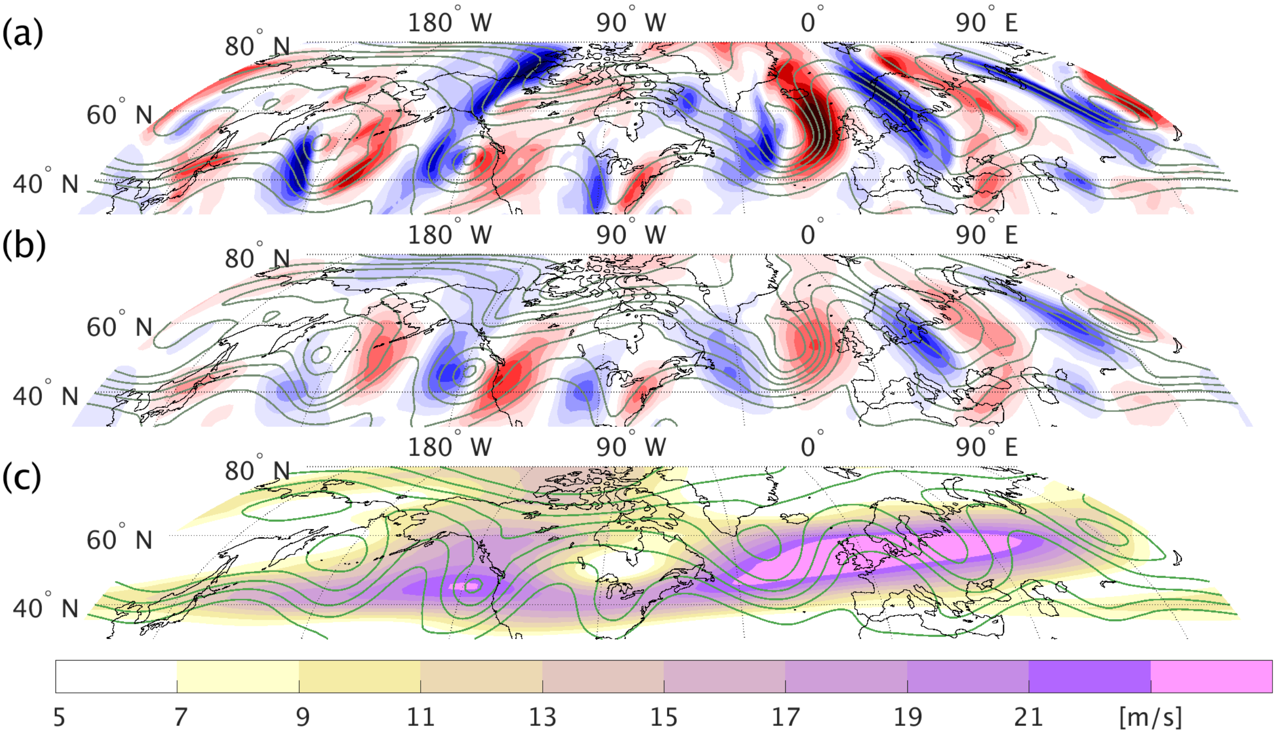

Figure 1: **Individual steps for the calculation of the quasi-stationary wave for the 09.08.2003, 00 UTC.** The panels show the following quantities: (a) meridional wind, (b) deviation of the 15 days lowpass filtered meridional wind from the daily climatology and (c) envelope field of the wind field shown in (b). Contour lines show geopotential between $8.8 \times 10^4\,\mathrm{m^2/s^2}$ and $9.5 \times 10^4\,\mathrm{m^2/s^2}$, separated by $10^3\,\mathrm{m^2/s^2}$. All variables are shown at 300 hPa.

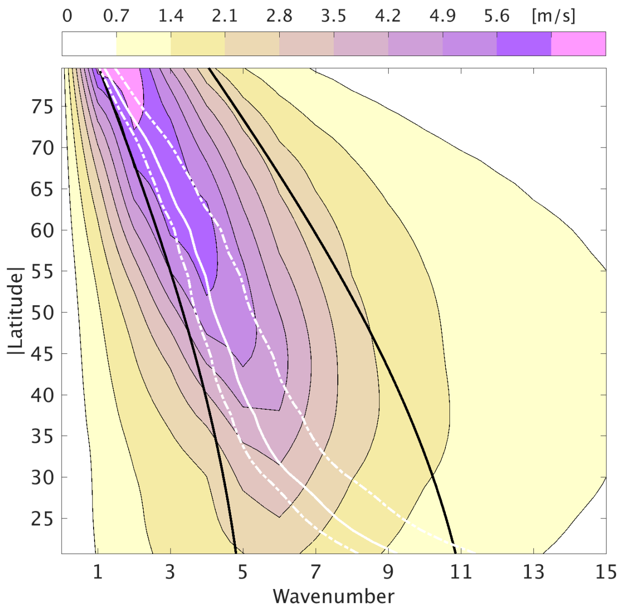

Figure 2: **Latitude dependent power spectra of the wind field used to calculate the QSW.** Shading shows the power spectra of the deviation of the 15 days lowpass filtered meridional wind from its daily-based climatological value at 300 hPa for the years 1980 until 2015. Black lines represent the latitude dependent wavenumber range used for the QSW calculation. White lines show the calculated wavenumbers by using the barotropic Rossby wave dispersion relation given in equation 1 with a meridional wavenumber $l = 2$, the zonal averaged climatological zonal wind and three possible phase speeds. The white solid (dashed) line(s) result from a phase speed equalizing 0.1 (0 and 0.5) of the strength of the zonally averaged climatological zonal wind.

equation (1) by k^{i}, with $k^{\mathrm{i}} = k\, a\, \cos(\varphi)$ (k^{i} is used for the x-axis in Fig. 2) and a the earth radius. By doing so we obtain

$$k^{\mathrm{i}}\left(u, c_{\mathrm{p}}, \varphi, l^{\mathrm{i}}\right) = a\, \cos(\varphi) \sqrt{\frac{2\,\Omega\, \cos(\varphi)/a - u_{\mathrm{yy}}}{(u - c_{\mathrm{p}})} - \left(\frac{l^{\mathrm{i}}}{a}\right)^2}\ , \tag{2}$$

where $c_{\mathrm{p}} = \omega/k$ is the zonal phase velocity and Ω is the angular velocity of Earth's rotation. Three possible scenarios of the function of k^{i} are given in Fig. 2 by the three white lines. We obtain these lines if we use for equation (2) a small meridional wavenumber ($l^{\mathrm{i}} = 2$), the zonal mean of the climatological zonal wind for u and a zonal phase velocity as fraction of u ($c_{\mathrm{p}} = 0.1\, u$ for the solid line and $c_{\mathrm{p}} = 0\, u$ and $0.5\, u$ for the two dashed lines).

This wavenumber range, given by the white lines, should represent the lower part of the chosen full wavenumber range (black lines). The input values for equation (2) are very rough estimates of what we expect. The white lines in Fig. 2 should motivate that with the chosen wavenumber range we are capturing the kind of waves we are interested in. For example, we chose the zonal mean of the zonal wind as background wind. But in the presence of a QSW, we expect lower values of the background wind, while everywhere else the values can be higher. If we would therefore divide u by 2 in equation (2), we would obtain higher values of k^{i}, capturing also more the right part of the wavenumber range (given by the black lines in Fig. 2). Lower phase speeds have a smaller impact and would shift the white lines slightly further to the left in Fig. 2, which is also the case for higher meridional wavenumbers, whereas lower zonal wind speeds lead to a stronger shift of the lines to the right into the higher wavenumber range inside the black lines. All in all, the considerations of this appendix should be sufficient to at least motivate that the chosen wavenumber range is reasonable.

References

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